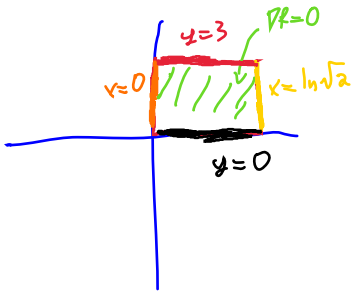
MTH241 Fall 2024: Quiz 06

Name: Hikaru no Go
UID:

Closed book, no calculator, show your work clearly.

1. (10pt) Find the extreme values of $f(x,y) = ye^{-2x}$ on the rectangular region R with vertices $(0,0), (\ln\sqrt{2}, 0), (\ln\sqrt{2}, 3)$ and $(0,3)$. (Grading: 3pt working; 1pt for critical point; 2pt for each boundary; 2pt for correct extreme values)



Interior: $\nabla f = \begin{bmatrix} -2ye^{-2x} \\ e^{-2x} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ But $e^{-2x} \neq 0$ for all x so no solution

Boundary: (1) $x=0$: $f(0,y) = ye^0 = y$, $0 \le y \le 3$ $\rightsquigarrow$ max $f = 3$, min $f = 0$

(2) $x = \ln\sqrt{2}$: $f(\ln\sqrt{2}, y) = ye^{-2\ln\sqrt{2}} = \frac{y}{2}$ $\rightsquigarrow$ max $f = \frac{3}{2}$, min $f = 0$

(3) $y=0$: $f(x,0) = 0$ $\rightsquigarrow$ max = min = 0

(4) $y=3$: $f(x,3) = 3e^{-2x}$ $\rightsquigarrow$ max $= 3e^0 = 3$, min $= 3e^{-2\ln\sqrt{2}} = \frac{3}{2}$

So the min of $f(x,y)$ is 0 at $\{(x,0) : 0 \le x \le \ln\sqrt{2}\}$

the max of $f(x,y)$ is 3 at $P = (0,3)$

Second page: